Mathematical Beauty—Enlightenment on Mathematics from "In Dialogue with Nature"

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1. Introduction

Of all intellectual fruits obtained by human intelligence, in my opinion, those in mathematics possess the highest positions, because mathematics is the philosophy of natural sciences, the core of Plato's eternal and perfect form of universe (Dunham 260). According to the "Preface to the Second Edition of *What is Mathematics?*", "mathematics hovers uneasily between the real and the not-real" and "links the abstract world of mental concepts to the real world of physical things without being located completely in either" (Stewart). This character gives mathematics incomparable beauty. In this course, texts on both natural sciences and mathematics have given me some enlightenment on mathematics, especially its seldom understood intellectual beauty. This is the core of this term paper.

2. Rigid Logic—the Base of the Mathematical System

As is known to all, mathematics feeds on logic, especially rigid logic. It seems always to be expressed in an extremely purely logical pattern, just like Euclid's *Elements*, "the greatest mathematical textbook of all time", where reason is in need even to demonstrate what "an ass knows by instinct" (Dunham 261, 272). The book is perfect in terms of logical expression, leaving no place for even the slightest change.

Without logic forming its bones, the whole mathematical system would collapse into nothing all of a sudden. It was Euclid's extreme logic that made his compass "collapsible" (Dunham 263–264), and it was also people's fear about the catastrophe of collapse that caused the "return to the classical ideal of precision and rigorous proof" in the 19th century (Courant and Robbins, "What is Mathematics?"). A mathematical system without logic can hardly survive, let alone grow overwhelming.

3. Beauty in More Intuitive Aspects

While some people have reached the agreement that ancient Greeks' rigid logic is beautiful as it built up a reasonable, neat and elegant system with only five postulates, and demonstrates the power of human mind (Dunham 272–273), many still remain unmoved. After all, rigid logic is often not intuitive enough to be related to beauty, and can sometimes be annoying.

However, quite different from the common view that "mathematics is nothing but a system of conclusions drawn from definitions and postulates that must be consistent but otherwise may be created by the free will of the mathematician" (Stewart, "Preface to the Second Edition of *What is Mathematics?*"), mathematics is much more than merely chains of rigorous logic. Were mathematics simply rigid logic, it would be able to develop simply by people's making all sorts of combinations of known mathematical entities in hope of coming across a valuable one, which is an unbearably boring process that a computer is capable to accomplish but no man is likely to spend time on (Poincaré 170). More often, extremely rigorous logic is unnecessary and can even hinder mathematical development. In the 17th century, a great new mathematical world was conquered with intuitive guesswork and blind confidence, that is, mathematicians depended hardly on logic in rigid form (Courant and Robbins, "What is Mathematics?"). What is more, there have been so many people devoting themselves to mathematics. They take pleasure in doing so because mathematics is in fact beautiful and fascinating intuitively.

3.1 Beauty and Generality

Beauty of mathematics lies in its generality.

Many general theorems like the Pythagorean Theorem are commonly regarded beautiful, because there are countless things in the real world. It is impossible to examine all of them respectively, so we hope to find some laws that can apply to all or at least most of them (Poincaré 161–162). Therefore, we find general laws useful, and thus beautiful (165–167).

Generality is part of intellectual and intuitive beauty also because generalisation is a tool that enables people to understand deeper what is already known. By generalisation, people get out of the original cave and see a much truer world (Plato 6–8), as generalisation in mathematics usually is adding in new concepts and methods while preserving the properties known in the original cave. For instance, starting from natural numbers, people defined rational numbers, real numbers and then complex numbers in succession by generalisation, each step preserving the algebraic properties of the former category of numbers.[1] The whole process brought humans better and deeper understanding of numbers and equations (Courant and Robbins 52–103).[2]

Mathematics itself is extraordinarily general. It is indisputable that all natural sciences and sometimes social sciences involve mathematics. For example, the geometrical representation resulted in a great leap in physics (Lindberg 35–44), and the introduction of mathematical analysis to physics was a great success as seen in *The Principia* (Cohen 49–59). In these two cases, physics was to a great extent related to quantity, and mathematics was an ideal tool to describe the relations between variables. Its presence in heredity also led Mendel to an incredible success. Here statistics, a branch of mathematics, revealed the existence of "specific factors" or genes (Watson 103–104). Mathematics is related to nearly everything, because what matters is not the actual meaning of numbers or points, but the structure and relationship between them (Courant and Robbins, "What is Mathematics?"). Thus a great variety of things have counterparts in mathematics when they come to relation. For example, calculus is used in both finance and design because of its general description of relation.

3.2 Beauty and Simplicity

Beauty of mathematics lies in its simplicity too.

Many simple things are considered beautiful because the real world around us is much too complicated, and we hope to somehow simplify it.

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1 For example, the addition and multiplication in the rational number field are the same when applied to integers. The same goes with complex numbers and real numbers (Courant and Robbins 52–103).

2 The introduction of complex numbers led to the discovery that every polynomial $f(x)$ with the highest power n can always be factorised into the product of exactly n factors $x-a_i$ $(i=1, 2, \ldots, n)$, where each a_i is a complex number (Courant and Robbins 101).

Hence, the laws describing the world need to be not only general but also simple. If we have general but very complicated laws, then they will have little chance to develop, because it takes hardly fewer efforts to analyse the laws than the real world.

Mathematics is usually simple, for it has been successfully developed. For example, Euclid's *Elements* is quite simple in that the construction of the whole system starts from extremely simple definitions and fairly self-evident postulates.[3] With all these few simple bricks, Euclid built an unbreakable mathematical sky-scraper using logic as his concrete. In addition, the proofs in the book, just like that of the proposition that in any triangle the sum of the length of two sides is larger than that of the third one, are quite simple and decent without losing rigid logic (Dunham 272). Another good example is the so-called "most beautiful equation", $e^{\pi i}+1=0$, which combines all important numbers so neatly and elegantly.[4]

Sciences and mathematics are simple also because scientists and mathematicians pursue beauty in simplicity, which is usually truth. During the pursuit, people may make wild guesses. When Watson and Crick were trying to find out DNA structure, unlike most chemists who considered it too complicated to be understood, they dared to go straight toward the simplest models, one of which turned out exactly true (Watson 123–133). Sometimes in mathematics, new concepts can even be invented for the sake of simplicity and beauty. The concepts of "ideal" things like the "ideal point" were invented

3 By "fairly", I mean these five postulates are actually not enough for Euclidean geometry, and one or two of the postulates can be slightly changed, leading to non-Euclidean geometries (Courant and Robbins 214–227).

4 This is a special case of Euler's Formula, $\cos x + i \sin x = e^{ix}$ (Courant and Robbins 477–479), where $x=\pi$. It should be noticed that 1, 0, e, π and i are all important numbers in mathematical history.

simply to avoid the annoying discussion concerning parallel lines.[5] After this invention, geometry, especially projective geometry, became simpler and more beautiful (Courant and Robbins 180–185).

3.3 Beauty and the Method of Making Discovery

Beauty of mathematics also lies in its method of making discovery.

The process of making mathematical discovery is pursuing mathematical beauty. According to Poincaré, the great mathematician, the process of making mathematical discovery usually involves three steps. The most amazing one, personally, is feeling the right combination in the subconscious ego and then letting it go into the conscious ego. This requires great sensibility to mathematical beauty (Poincaré 170–178). In other words, this process is just in search for a beautiful combination intuitively.

What is more, this method is interesting and beautiful itself. Working hard day and night, according to Poincaré and my personal experience as well, is no more efficient than working unconsciously (Poincaré 171–178). Compared with the former, the latter method is much more elegant and similar to art's style. This method is not all about hard work, but about intuition, inspiration and sensibility to beauty. On the other hand, the process of subconscious discovery is just like a jigsaw puzzle, where the player looks for a right combination day and night unconsciously and intuitively from an aesthetic and logical standpoint, and finally carefully puts together the pieces selected by subconscious ego to complete his ideal work.

5 An "ideal point" is the point where all lines parallel to each other intersect. Similarly, an "ideal line" is the line where all planes parallel to each other intersect (Courant and Robbins 180–185).

4. Conclusion

Richard Courant and Herbert Robbins pointed it out that "[m]athematics as an expression of the human mind reflects the active will, the contemplative reason, and the desire for aesthetic perfection" ("What is Mathematics?"). It is not totally about reality, but allows for creation and imagination. It is not totally about abstract signals, but an attractive game where participants try both intuitively and logically to piece together the given pieces in order to create a beautiful work. It is a perfect mixture of science and philosophy, rationality and intuition, precision and art, rigidity and beauty. All these properties make mathematics logical, general, simple, interesting and, as a result, beautiful. It is hard to play this game well, but it is still fascinating and satisfactory enough merely to play it.

Works Cited

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Teacher's comment:

Based on the texts of "In Dialogue with Nature", Hongxiang discusses the beauty of mathematics from three aspects: generality, simplicity, and elegance of methodology. It is nicely presented that mathematics is a prefect mixture of science and philosophy, rationality and intuition, precision and art, rigidity and beauty. This article is well-organised and the whole story is vividly told, making the reading experience quite joyful. (Yang Jie)