Absolute Beauty: The Face of Phi Φ

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Beauty is eternity gazing at itself in the mirror.

— Kahlil Gibran, *The Prophet*[1]

What is Beauty?

Throughout history, humankind has been obsessed with beauty. Beauty serves as an object of reverence, an impetus for action, a source of inspiration, and a prize to be sought after. Mythology tells of how Helen of Troy's face had a beauty that set a thousand ships in motion and stimulated a war. Scriptures reveal how the Jewess Hadassah had a beauty that crowned her as Queen Esther of Persia. The night sky displays stars with a beauty that draws the astronomer to scrutinize and unite them into constellations. Nature parades sunsets with a beauty that inspires the artist to produce magnificent creation. The power of beauty is ubiquitously visible: in humans, it aids the celebrity's rise to fame and confers advantageous salaries[2] while in buildings[3] and nature, it draws admiring travelers from distant locations. The questions then remain: what is beauty? Does beauty have a scientific basis or is it merely an aesthetic

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1   K. Gibran, *The Prophet* (New York: Alfred A. Knopf Inc., 1923).
2   "Over his career, a good-looking man will make some $250,000 more than his least-attractive counterpart." D. Hamermesh. "The Beauty Advantage". *Newsweek*. 19 July 2010. Web version: http://www.newsweek.com/2010/07/19/the-beauty-advantage.html.
3   An example would be St. Peter's Basilica, which annually receives 18 million visitors. Rk4u. *Top Religious Destinations in the World*. 08 July 2009. http://www.indiancinemafans.com.

whim of human fancy? This paper first expounds on Henri Poincaré's idea of intellectual beauty, then challenges his claim that his theory is inapplicable to visual beauty through mathematical investigation.

Beauty of Science

Henry Poincaré claimed that

> The scientist does not study nature because it is useful to do so. He studies it because he takes pleasure in it and he takes pleasure in it because it is beautiful. If nature were not beautiful, it would not be worth knowing, and life would not be worth living.[4]

Therefore, the impetus for the scientific search for facts, the source of scientific pleasure, and the meaning of life are all found in beauty. The goal of science is not to "know all the facts," but rather, to sift through and select, from an infinite quantity of facts, the ones that are the most intellectually beautiful. However, since scientific discovery is often guided by utility rather than loveliness, do these intellectually exquisite facts coincide with the most valuable facts? Poincaré argues that they concur, for the most useful facts are equivalent to "those which have a chance of recurring" and repetition occurs most often in simple facts, which are found "in the two extremes, in the infinitely great and the infinitely small" and whose simplicity renders them to be orderly and harmonious, which manifests as beauty to the scientific mind.[5]

4　H. Poincaré, "Science and Method" *in The Value of Science: Essential Writings of Henri Poincaré.* (New York: The Modern Library 2001), p. 368.

5　*Ibid.*, p. 366.

Mathematics of Beauty

Now can Poincaré's theory of beauty as a simple, orderly, recurrent pattern be applied to physical as well as intellectual beauty? Poincare claims that "the beauty of qualities and appearances…have nothing to do with science"[6]. But what if there is a scientific foundation, or persistent mathematical formula that defines loveliness? Is there an objective measure to beauty? The answers lie hidden in Φ (Phi), the Golden Mean.

The Fibonacci Sequence is a series of integers starting with 1, 1, 2, 3, 5, 8, 13, 21, 34, 55… that continues infinitely, and is defined by the subsequent number being the sum of the two previous numbers. If the ratio of adjacent numbers are taken[7]: 1/1=1, 2/1=2, 3/2=1.5, 5/3=1.66, 8/5=1.6, 13/8=1.625, 21/13=1.615, it is found that as the numerators and denominators increase in value, the ratios begin to converge at the irrational number of approximately 1.618033989, which is Φ or the Golden Mean.

This Golden Mean's versatility allows its ubiquitous presence to be seen in architecture, art, the solar system, plants, animals, and even humans[8]. Wherever it is found, it is deemed as something beautiful and appealing to the eye. The first example is seen in the Φ (Phi) Rectangle. If the numbers of the Fibonacci are used as the sides of a series of squares, where each successive square is added to the longer side, the width-length ratio will draw ever nearer to Φ, or 1.618033989 (Figure 1).

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6 *Ibid.,* p 368.
7 The succeeding number serves as the numerator while the preceding number serves as the denominator.
8 Oracle: Think Quest. "The Beauty of the Golden Ratio", 2005. Web version: http://library. thinkquest.org/trio/TTQ05063/phibeauty1.htm.
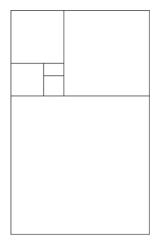
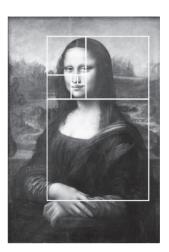*Figure 1*. The Fibonacci
Sequence in the Φ Rectangle
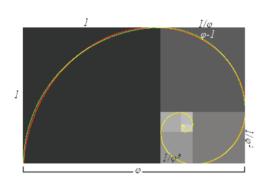
Figure 2. The Φ Rectangle in
the Mona Lisa

This Phi Rectangle has been incorporated into Leonardo Da Vinci's Mona Lisa, which has been hailed as the most beautiful painting in the world. The woman's focal points, like her elbow and wrist, form the corners of the rectangle, while her chin, the top of her head, her nose, and her mouth are all aligned with sides of the smaller Fibonacci-unit squares[9].
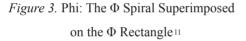
A second example is the Φ Spiral, or Equiangular Spiral, which is a curve superimposed on the Φ Rectangle so that the curve intersects the sides of the rectangle at the juncture between component squares (Figure 3)[10]. This spiral is called Equiangular, as any point drawn from the center to intersect the sides of the spiral will form an 80º angle. In nature, perhaps the most perfect example of this Φ Spiral is found in the nautilus, but other examples

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9   S. Obara. "Golden Ratio in Art and Architecture". http://jwilson.coe.uga.edu/EMT668/ EMAT6680.2000/Obara/Emat6690/Golden%20Ratio/golden.html.

10   The equation for the Equiangular Spiral is $r = ae\theta \cot b$

include spiral arms of galaxies, the cloud pattern of typhoons, and the curl of the fern (Figure 4). True to Poincaré's claim, wherever this simple, repeating pattern has been found, it confers beauty upon its subject.
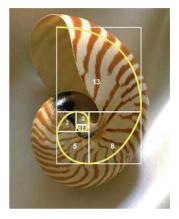

Figure 3. Phi: The Φ Spiral Superimposed on the Φ Rectangle[11]

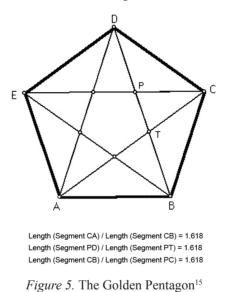*Figure 4.* The Nautilus Shell[12]

"Love of Beauty is Taste" claims Ralph Waldo Emerson, echoing belief that the beauty of a human face is a matter of subjective taste. However, is beauty truly subjective? Since research of multiple cultures indicates that there is common consensus on the rating of facial beauty, this seems to be an indication of recurrent underlying factors[13]. Could it be that human beauty is not appraised by fanciful whims, but rather, determined by a scientific basis? The answer lies, once again, in the Golden Mean.

11   Image by Pau, available under the Creative Commons Attribution-Share Alike 3.0 Unported license. Image URL: http://en.wikipedia.org/wiki/File:FakeRealLogSprial.svg (Accessed 20/9/2011).

12   Author unknown, available under GNU Free Documentation License. Image URL: http://mathforum.org/mathimages/index.php/Image:Shell.jpg (Accessed 20/9/2011).

13   M. Bashour. *An Objective System for measuring Facial Attractiveness*. Plastic and Reconstructive Surgery. 1 September 2006; 118(3): 757–774.

Connecting two Φ lines of 1.618 units and one 1.000 unit line creates an isosceles triangle called the Acute Golden Triangle. Adding two Obtuse Golden Triangles (with two 1.000 unit lines and one 1.618 line) creates the Golden Pentagon (Figure 5). Duplicating the Golden Pentagon and rotating it 36º creates the Golden Double Star Complex, whose vertices can be connected to from The Golden Decagon Matrix, or the Φ Matrix (Figure 6). The superimposition of a cross section of DNA revels that it has a similar configuration to that of the Golden Decagon Matrix[14].

Length (Segment CA) / Length (Segment CB) = 1.618
Length (Segment PD) / Length (Segment PT) = 1.618
Length (Segment CB) / Length (Segment PC) = 1.618

Figure 5. The Golden Pentagon[15]

Poincaré claimed that a useful, beautiful fact was one that had "a high chance of recurring"[16]. What is more often repeated than DNA, the code of

14　S. Marquardt. Marquardt Beauty Analysis. Web version: http://www.beautyanalysis.com/.
15　Adapted from an online image, title and author unknown. Image URL: http://jwilson.coe. uga.edu/EMT668/EMAT6680.2000/Obara/Emat6690/Golden%20Ratio/image46.gif.
16　H. Poincaré, "Science and Method" *in The Value of Science: Essential Writings of Henri Poincaré.* (New York: The Modern Library 2001), p. 364.

biological life? Moreover, Poincaré's beautiful facts must be simple in that they are "infinitely small" to the point that the distinctive features between organisms are replaced by the homogenous microscopic building blocks. DNA fits this description perfectly.
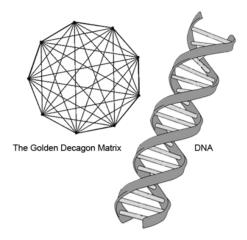

The Golden Decagon Matrix   DNA

Figure 6. The Golden Decagon Matrix and DNA

However, the application of the Φ-derived Golden Decagon Matrix does not end there. Using line segments of the Φ Matrix combined with 42 secondary Φ Matrices, Stephen Marquardt created the Φ Mask, which outlines the geometrically perfect face[17]. Subsequent research demonstrated the level of correlation between a human face and the Φ Mask was directly proportional to the level of perceived attractiveness. From the proportion of the eyes to the mouth to the ratio of the width of the brow to the bridge of the nose, every line screams one number: 1.618. The Golden Mean. The number of beauty.

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17   The "Marquardt Beauty Analysis" website provides images of such application. See 14.

Conclusion: Beauty is Eternity

The Golden Mean is the marriage between science and beauty, providing a mathematical explanation and objective basis for an aesthetic idea. Though Poincaré was correct in claiming that a scientist "takes pleasure in [his studies] because it is beautiful," he erred in the belief that the "beauty which strikes the senses . . . the beauty of qualities and appearances . . . has nothing to do with science"[18]. Poincare lucidly comprehended the beauty of math and science but had yet to conceive of the math and science of beauty. What is beauty? It is the curl of the fern's frond, the foundations of the Gizaian Pyramids,[19] the body of The Vetruvian Man,[20] and the face of Aishwarya Rai[21], all of which share one quantifiable and qualifiable trait: the Golden Mean. What is Φ? Not only is it the most attractive number in the world, but it is the essence of beauty itself. Beauty is 1.618033989… trailing off into infinity. Truly, "Beauty is eternity gazing at itself in the mirror"[22].

References

1.　　K. Gibran, *The Prophet*. (New York: Alfred A. Knopf Inc., 1923).

2.　　D. Hamermesh. *The Beauty Advantage*.　19 July 2010. Web version: http://www.newsweek.com/2010/07/19/the-beauty-advantage.html.

3.　　Rk4u. *Top Religious Destinations in the world*. 08 July 2009. http://www.

18　H. Poincaré, "Science and Method" *in The Value of Science: Essential Writings of Henri Poincaré*. (New York: The Modern Library 2001), p. 364.

19　The length of the pyramid's base is 756 feet while the height is 481 feet, and the ratio of 756 to 481 is Φ.

20　A painting by Leonardo Da Vinci whose body proportions (arms, legs, torso) reflected Φ.

21　An Indian actress who is consider by many to be the most beautiful woman in the world.

22　K. Gibran, *The Prophet*. (New York: Alfred A. Knopf Inc., 1923).

indiancinemafans.com/board/upload/story-pics-111/top-religious-tourist-destinations-world-45277/.

4.   H. Poincaré, "Science and Method" in *The Value of Science: Essential Writings of Henri Poincaré*. (New York: The Modern Library 2001).

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7.   S. Marquardt. *Marquardt Beauty Analysis*. Web version: http://www.beautyanalysis.com/.

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9.   M. Bourne. *Equiangular Triangle*. 11 February 2011. Web Version: http://www.intmath.com/plane-analytic-geometry/equiangular-spiral.php.

10. B. Safran. *The Mathematics of Beauty: The Divine Proportion's Effect on Facial Attractiveness*. 2007. Brown University: Providence. Web Version: http://www.brown.edu/Departments/Economics/The%20Mathematics%20of%20Beauty%20-%20Ben%20Safran.pdf.

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Teacher's comments:

The writer performed very well in sorting out various examples about the Golden ratio (Phi) and related them into the human sense of beauty. Such relation was established by the equivalence of beauty and eternity. Concepts

were explained clearly and the article is well organized. (Chan Chi Wang, Ng Wai Yin, Szeto Wai Man and Wong Wing Hung)